

BASICAL DIMENSIONS OF CLASSIFYING CRYSTALLIZERS

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Relations for calculation of basic dimensions of crystallizers operated with perfect classification of suspension were derived on basis of material and number of crystals balances. Relations thus obtained were compared with equations published earlier which were derived under very simplified conditions. Dependence of basic dimensions of crystallizers on crystallization parameters is discussed.

Continuous classifying crystallizers are widely used for production of a number of products since they provide large, well developed crystals with a narrow range of size distribution. Despite their relatively common use they are usually designed on empirical basis and the quantitative description of the crystallization kinetics is seldom used in determination of their basic dimensions.

The literature offers a number of works dealing with the problematics of classifying crystallizers. Let us mention first of all the basic work by Miller and Seaman¹, referred to in most of works on classifying crystallizers. In this work the authors have derived relations for calculation of the size distribution of crystals and of supersaturation along the height of the fluidized bed and verified validity of these relations on a pilot-plant model with crystallization of ammonium nitrate. Another significant contribution to the theory of classifying crystallizers was made by Bransom^{2,3} who pointed, among other, to the importance of knowledge of the crystallization kinetics for calculation of classifying crystallizers and drew the attention to the fact that the supersaturation of the solution leaving the fluidized bed of crystals could quite differ from the earlier considered zero supersaturation, and that the volume of the crystallizer depends very much on the degree of desupersaturation of the solution. Mathematical model of a classifying crystallizer⁴ based on assumption of complete desupersaturation was, therefore, generalized for the case when a significantly supersaturated solution⁵ still leaves the fluidized bed. There was also pointed out that for each system can be found a certain optimum size of a classifying crystallizer due to the existing dependences between individual parameters. Finally let us mention the work of Pulley⁶ which deals in detail with the density of suspension in the fluidized bed and which gives for porosity in the bottom and top parts of the fluidized bed theoretical values $\epsilon = 0.5$ and $\epsilon = 0.975$.

None of the mentioned works deals in detail with the balance of number of crystals; this is caused by the fact that the nucleation process which takes place in the classifying crystallizer is very complex and is complicated by a changing supersaturation at circulation of the solution as well as by a variable content of solid phase in the suspension.

Nor the calculation of main parameters of the crystallizer has been explicitly derived for the case of incomplete desupersaturation. The mentioned problems are the subject of this study.

THEORETICAL

It is obvious that in a continuously operating crystallizer the number of originating respectively fed crystals must equal the number of product crystals plus the number of crystals removed. In a classifying crystallizer is this balance of number of crystals balanced by the crystal nuclei originated in a slight excess and this excess of small crystals is either led to the dissolving vessel or is directly dissolved in a circulation loop.

In the ideal state would, however, originate just the necessary number of crystal nuclei.

When we express the nucleation rate by a semiempirical equation⁷

$$dm_z/dt = k_n(\Delta c)^m \quad (1)$$

then the number of crystals originated in a unit of solution in a unit of time can be calculated from the relation

$$N_z = k_n(\Delta c)^m / \alpha Q_c L_z^3. \quad (2)$$

However, in a classifying crystallizer the supersaturation is not constant but it decreases during the passage through the fluidized bed of crystals from the initial value Δc_p down to the final value Δc_0 . For this passage of solution through the fluidized bed must be valid the materials balance

$$Q(\Delta c_p - \Delta c_0) = P[1 - (L_0^3/L_p^3)]. \quad (3)$$

When we define the degree of desupersaturation as⁵

$$\gamma = 1 - \Delta c_0/\Delta c_p \quad (4)$$

we can write

$$\gamma = (P/Q \Delta c_p) (1 - L_0^3/L_p^3). \quad (5)$$

In order to be able to calculate the number of nuclei at the variable supersaturation, we shall first determine the mean value of supersaturation with regard to nucleation rate:

$$\begin{aligned} \overline{(\Delta c)_n}^m &= [1/(\Delta c_p - \Delta c_0)] \int_{\Delta c_0}^{\Delta c_p} (\Delta c)^m d(\Delta c) = \\ &= [(\Delta c_p)^m / (m + 1) \gamma] [1 - (1 - \gamma)^{m+1}] \end{aligned} \quad (6)$$

or

$$\overline{(\Delta c)_n}^m / (\Delta c_p)^m = \Gamma, \quad (7)$$

where

$$\Gamma = [1 - (1 - \gamma)^{m+1}] / (m + 1) \gamma. \quad (8)$$

Values of Γ can be, as a function of γ and m , read off the Fig. 1. In Fig. 2 are as a function of γ and m plotted the values $\overline{\Delta c} / \Delta c_p$, calculated from Eq. (7). The balance of number of crystals for the whole volume of the fluidized bed can be written in the form

$$[k_n \overline{(\Delta c)_n}^m / \alpha \rho_c L_z^3] W_0 = P / \alpha \rho_c L_p^3 \quad (9)$$

or

$$[k_n (\Delta c_p)^m / \alpha \rho_c L_z^3] \Gamma W_0 = P / \alpha \rho_c L_p^3. \quad (10)$$

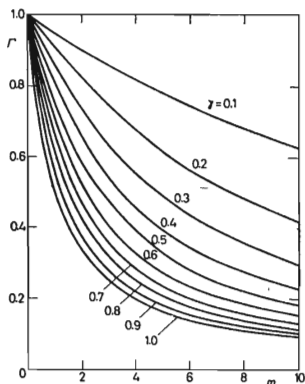


FIG. 1

Values of Function Γ Defined by Eq. (8)

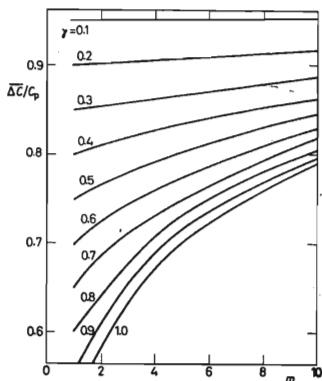


FIG. 2

Average Supersaturation in Fluidized Bed Calculated with Regard to Nucleation

Eq. (9) resp. (10) assume that the nuclei originate only in the fluidized bed. With regard to the metastable region being in presence of crystals considerably narrower than in the clear solution⁷, this assumption can be considered as reasonable.

From Eq. (10) we can then calculate the initial supersaturation of the inlet solution Δc_p which should provide sufficient number of nuclei for a continuous operation of the crystallizer according to

$$(\Delta c_p)^m = PL_z^3/k_n \Gamma W_0 L_p^3 = PL_z^3(c+1)/k_n \Gamma V \bar{\varepsilon} \rho_c L_p^3, \quad (11)$$

while the concentration values c and densities ρ_l of solution are taken as an average of the inlet and outlet values.

In one of previous papers⁵ was derived the following equation for the growth of crystals

$$dL/dt = g_p \{1 - \gamma[1 - (L^3/L_p^3)]\}^n, \quad (12)$$

where the linear crystallization rate g_p is given by relation

$$g_p = k_g \beta (\Delta c_p)^n / 3\alpha \rho_c. \quad (13)$$

A general solution of the differential Eq. (12) leads to complicated relations which can be formally simplified into the form⁵

$$\tau = CL_p/g_p, \quad (14)$$

while the value of the proportionality constant C is a function of the order of the kinetic equation of the crystals growth n , of the value γ , and of the value L_0/L_p . The results of calculation of the constant C made for selected parameter values on a digital computer, are given in Table I.

The relation for the amount of the solid phase contained in the crystallizer was derived in the same work⁵ in the form:

$$W = (1/4)P\tau L_p/(L_p - L_0), \quad (15)$$

which with the use of Eq. (13) and (14) gives

$$W = \{3\alpha \rho_c L_p P C / [4k_g \beta (\Delta c_p)^n]\} [L_p/(L_p - L_0)]. \quad (16)$$

When we assume on the basis of data provided by Pulley⁶, that the mean porosity of the fluidized bed⁵ is $\bar{\varepsilon} = 0.85$ or that

$$1 - \bar{\varepsilon} = W/(V\rho_c) = 0.15, \quad (17)$$

we can express the volume of the fluidized bed by the relation

$$V = 5\alpha L_p P C / [k_g \beta (\Delta c_p)^n] [L_p/(L_p - L_0)] \quad (18a)$$

or from Eq. (11) by

$$V = P(c+1)/[\rho_l k_n \Gamma \bar{\varepsilon} (\Delta c_p)^m] (L_z/L_p)^3. \quad (18b)$$

TABLE I
Values of Proportionality Constant C in Eq. (14)

n	γ	L_0/L_p						
		0.9	0.8	0.7	0.6	0.5	0.4	0.3
1.0	1.0	0.117	0.281	0.520	0.889	1.500	2.625	5.056
	0.8	0.113	0.258	0.445	0.683	0.984	1.350	1.776
	0.6	0.110	0.240	0.393	0.570	0.770	0.990	1.224
	0.4	0.106	0.224	0.354	0.495	0.645	0.802	0.964
	0.2	0.103	0.211	0.324	0.441	0.561	0.683	0.807
1.2	1.0	0.121	0.302	0.588	1.067	1.947	3.781	8.417
	0.8	0.116	0.273	0.484	0.768	1.143	1.617	2.187
	0.6	0.112	0.249	0.416	0.614	0.844	1.101	1.379
	0.4	0.107	0.230	0.367	0.518	0.680	0.852	1.030
	0.2	0.104	0.214	0.330	0.450	0.575	0.702	0.831
1.4	1.0	0.125	0.326	0.666	1.290	2.559	5.553	14.41
	0.8	0.199	0.288	0.527	0.866	1.333	1.949	2.710
	0.6	0.114	0.258	0.440	0.663	0.927	1.228	1.558
	0.4	0.109	0.235	0.380	0.541	0.717	0.906	1.102
	0.2	0.104	0.216	0.335	0.459	0.588	0.721	0.855
1.6	1.0	0.130	0.351	0.757	1.570	3.402	8.296	25.28
	0.8	0.122	0.304	0.576	0.979	1.561	2.360	3.378
	0.6	0.116	0.269	0.467	0.716	1.019	1.372	1.763
	0.4	0.110	0.241	0.393	0.566	0.757	0.963	1.179
	0.2	0.105	0.218	0.340	0.469	0.602	0.740	0.881
1.8	1.0	0.134	0.379	0.865	1.924	4.571	12.58	45.20
	0.8	0.126	0.322	0.630	1.110	1.836	2.873	4.233
	0.6	0.118	0.279	0.495	0.775	1.123	1.536	2.000
	0.4	0.111	0.247	0.407	0.592	0.800	1.025	1.263
	0.2	0.105	0.221	0.346	0.478	0.617	0.760	0.907
2.0	1.0	0.139	0.410	0.990	2.372	6.200	19.33	82.10
	0.8	0.129	0.340	0.690	1.262	2.168	3.513	5.331
	0.6	0.120	0.290	0.525	0.839	1.239	1.722	2.273
	0.4	0.113	0.253	0.422	0.620	0.845	1.092	1.354
	0.2	0.106	0.223	0.351	0.488	0.632	0.781	0.934
2.5	1.0	0.151	0.502	1.409	4.103	13.77	59.25	385.1
	0.8	0.138	0.394	0.873	1.761	3.337	5.916	9.673
	0.6	0.126	0.320	0.612	1.030	1.596	2.313	3.156
	0.4	0.116	0.268	0.461	0.697	0.972	1.281	1.615
	0.2	0.108	0.230	0.366	0.514	0.671	0.836	1.006

TABLE I
(Continued)

0.25	0.20	0.15	0.10	0.09	0.08	0.07	0.06	0.05
7.500	12.00	21.72	49.50	61.23	77.63	101.5	138.4	199.5
2.007	2.246	2.490	2.738	2.788	2.838	2.888	2.938	2.988
1.345	1.468	1.592	1.717	1.742	1.767	1.792	1.817	1.842
1.046	1.129	1.212	1.295	1.312	1.329	1.345	1.362	1.379
0.869	0.932	0.994	1.056	1.069	1.081	1.094	1.106	1.119
13.75	24.87	52.97	152.7	201.0	273.1	386.7	577.5	928.1
2.500	2.827	3.163	3.505	3.573	3.642	3.711	3.780	3.849
1.524	1.671	1.820	1.969	1.999	2.029	2.059	2.089	2.119
1.121	1.212	1.304	1.397	1.415	1.433	1.452	1.470	1.489
0.896	0.961	1.026	1.091	1.104	1.117	1.131	1.144	1.157
26.08	53.58	135.0	495.0	693.6	1 011	1 551	2 540	4 552
3.135	3.582	4.044	4.514	4.609	4.704	4.799	4.894	4.989
1.730	1.907	2.085	2.264	2.300	2.336	2.372	2.408	2.445
1.202	1.303	1.405	1.507	1.527	1.548	1.568	1.589	1.609
0.923	0.991	1.059	1.128	1.141	1.155	1.169	1.182	1.196
50.79	118.9	355.4	1 660	2 478	3 877	6 440	11 570	23 140
3.955	4.566	5.200	5.848	5.979	6.110	6.241	6.372	6.503
1.969	2.180	2.394	2.609	2.653	2.696	2.739	2.782	2.826
1.290	1.402	1.514	1.627	1.650	1.673	1.695	1.718	1.741
0.951	1.023	1.094	1.165	1.180	1.194	1.208	1.222	1.237
101.1	270.2	958.6	5 709	9 076	15 240	27 430	54 050	1.206/5
5.017	5.852	6.724	7.616	7.797	7.977	8.158	8.338	8.519
2.246	2.498	2.755	3.013	3.065	3.117	3.169	3.221	3.273
1.385	1.509	1.634	1.759	1.784	1.809	1.834	1.859	1.884
0.981	1.055	1.130	1.204	1.219	1.234	1.249	1.264	1.279
204.6	624.8	2 634	20 000	33 870	61 040	1.190/5	2.573/5	6.406/5
6.395	7.538	8.735	9.965	10.21	10.46	10.71	10.96	11.21
2.567	2.869	3.176	3.487	3.549	3.611	3.674	3.736	3.799
1.489	1.626	1.764	1.902	1.930	1.958	1.985	2.013	2.041
1.011	1.069	1.167	1.245	1.261	1.276	1.292	1.307	1.323
1 260	5 375	34 870	4.866/5	9.652/5	2.076/6	4.945/6	1.347/7	4.412/7
11.96	14.46	17.11	19.84	20.40	20.95	21.51	22.07	22.63
3.614	4.087	4.571	5.062	5.160	5.259	5.357	5.456	5.555
1.788	1.964	2.142	2.321	2.356	2.392	2.428	2.464	2.500
1.092	1.179	1.266	1.353	1.371	1.388	1.406	1.423	1.441

TABLE I
(Continued)

n	γ	L_0/L_p						
		0.9	0.8	0.7	0.6	0.5	0.4	0.3
3.0	1.0	0.165	0.620	2.043	7.318	31.88	190.7	1 905
	0.8	0.147	0.457	1.116	2.496	5.239	10.19	17.95
	0.6	0.132	0.355	0.716	1.273	2.075	3.137	4.430
	0.4	0.120	0.285	0.506	0.785	1.123	1.510	1.935
	0.2	0.109	0.236	0.381	0.541	0.713	0.896	1.085

$$x/y = x \cdot 10^y.$$

By eliminating Δc_p from these two relationships we get

$$\left(\frac{V}{P}\right)^{1-n/m} = \frac{5}{3} \frac{C}{BQ_c} \left(\frac{\Gamma \bar{e} Q_1}{c+1}\right)^{n/m} \left(\frac{L_p}{L_p - L_0}\right) \left(\frac{L_p}{L_z}\right)^{1+3n/m}, \quad (19)$$

where appears the system constant B , given by

$$B = k_g \beta / 3 \alpha Q_c k_n^{n/m} L_z, \quad (20)$$

which is the same one as in the case of batch or continuous agitated crystallizer⁸. This enables to make the calculation of the classifying crystallizer either by use of the experimentally found kinetic data on crystal nucleation and growth or of the constant B determined for example from a suitably programmed discontinuous experiment in the agitated crystallizer⁸.

If we substitute Eq. (18) into Eq. (11) we get the expression for calculating the suitable inlet supersaturation

$$(\Delta c_p)^{m-n} = \frac{k_g \beta (c+1)}{4.25 k_n \Gamma C Q_1} \frac{L_z^3 (L_p - L_0)}{L_p^5}, \quad (21)$$

For calculation of the free cross-sectional area of the crystallizer from the material balance (5) holds

$$A = Q(1+c)/\varrho_1 = [P(1+c)/u\gamma \Delta c_p \varrho_1] [1 - (L_0^3/L_p^3)] \quad (22)$$

TABLE I
(Continued)

0.25	0.20	0.15	0.10	0.09	0.08	0.07	0.06	0.05
8 192	48 830	4.878/5	1.250/7	2.905/7	7.455/7	2.171/8	7.456/8	3.212/9
22.87	28.33	34.19	40.29	41.52	42.76	44.01	45.25	46.50
5.142	5.885	6.647	7.421	7.577	7.733	7.889	8.045	8.201
2.157	2.383	2.612	2.843	2.889	2.935	2.981	3.028	3.074
1.181	1.278	1.375	1.473	1.492	1.512	1.531	1.551	1.570

and for the height of the fluidized bed

$$h = \frac{V}{A} = \frac{5\alpha C u \gamma \rho_1}{k_g \beta (\Delta c_p)^{n-1} (1+c)} \frac{L_p^2}{(L_p - L_0) (1 - L_0^3/L_p^3)} \quad (23)$$

The necessary amount of circulating liquid can be calculated by use of the modified Eq. (5) which is

$$V_Q = [P(1+c)/\gamma \Delta c_p \rho_1] [1 - (L_0^3/L_p^3)] \quad (24)$$

DISCUSSION

In the first place it is necessary to realize the initial assumptions under which the calculation has been done and which will further on limit the validity of resulting relationships:

1. Kinetics of nucleation in the suspension of crystals is described with sufficient accuracy by equation of the type (1). As long as other forms of nucleation do not take place for example by attrition of crystals, this assumption can be accepted.
2. Kinetic parameters of nucleation measured in laboratory are identical with corresponding values describing the nucleation in an industrial crystallizer. On the basis of experience can be assumed that the mentioned condition is quite reliably valid for the nucleation order m . The value of kinetic constant k_n is, however, very sensitive to a great number of parameters characterizing the experimental conditions and therefore, the direct use of values measured in the laboratory may not be very reliable. Fortunately, the value of this constant usually appears in calculations in the form $k_n^{n/m}$, and since usually $n < m$, the error caused by the use of laboratory data will not be significant. Still more accurate calculation can be obtained when we use Eq.

(19) with the value of constant B calculated not from partial data (Eq. 20) but directly from its values experimentally determined by independent measurement⁸. 3. Growth rate of crystals in suspension is described by a kinetic equation

$$dm_c/dt = k_g A_1 \Delta c^n. \quad (25)$$

Despite of some opinions (*e.g.*²) that the growth rate of crystals of some materials depends also on their size, the mentioned equation is generally accepted. 4. The kinetic constants of crystal growth measured in laboratory can be used as well for the case of suspension in an industrial crystallizer. This assumption can be accepted on the basis of so far published data. 5. At a steady operation of the crystallizer, the supersaturation of the solution will reach such a value that the number of nuclei will exactly equal to the number of product crystals removed. If this condition is satisfied we can use for calculation of volume of the fluidized bed Eq. (19) while the steady value of supersaturation can be calculated from Eq. (21). In practice, the steady operation of the crystallizer is usually attained in such a way, that the nuclei in excess are removed and dissolved. In this case Eq. (21) and (19) cannot be used and the volume of the crystallizer is calculated from the real value of inlet supersaturation according to Eq. (18). 6. The fluidized bed of crystals is perfectly classified, *i.e.* in each horizontal level are only crystals of the same size. This fact does not, however, correspond to actual situation existing in the classifying crystallizer where the crystals of various sizes are partially mixed together. Nevertheless, this idealizing condition is currently accepted¹⁻⁶; deviations of reality from this idealized model are partially compensated by both smaller and larger crystals than expected present, due to mixing in each horizontal layer. 7. Deviations in crystal shapes from an isotropic body (cube) are sufficiently characterized by geometric factors

$$\alpha = (\text{crystal volume})/L^3 \quad (26)$$

and

$$\beta = (\text{surface area of a crystal})/L^2. \quad (27)$$

This assumption is generally accepted in literature. 8. The crystallizer operates in a steady state. This is a condition for derivation of a simple mathematical model, but it need not be always satisfied in reality^{9,10} in which case the crystallizer operates in more or less pronounced cycles.

There is a whole number of limiting factors and from that follows the necessity to carry out the calculation very carefully and sensibly and to check and eventually correct all partial results according to practical experience. Under such conditions it is possible to get relatively reliable results.

CALCULATIONS

The most suitable calculation procedure can be characterized by the following scheme:

I. *Kinetic parameters of nucleation and of the crystal growth are known.* 1. from Eq. (20) we calculate the value of the system constant B . 2. We select the value L_0 and the corresponding upflow rate u ; literature^{2,6} gives usually $L_0/L_p \geq 0.1$ but it rather depends on physical properties of the system and on the crystal shapes. For a better approach it is necessary to make either experimental measurements of conditions suitable for fluidization or to use some of semiempirical correlations, for example^{2,11}

$$Le^2/[u(1 - \epsilon)^{1/3}] = \text{const.} \quad (28)$$

or other analogical relations¹² between L and u .

3. We choose a number of degrees of desupersaturation γ and from Fig. 1 we read off the corresponding values Γ . From Table I we take the value of constant C . 4. From Eq. (19) we calculate volume of the fluidized bed. 5. The partial result is checked: the value of separation intensity defined¹³ as

$$SI = 3.6 \cdot 10^6 LP/V, \quad (29)$$

should be about 100, more probably less since the specific output of classifying crystallizers is used to be somewhat lower than that of agitated crystallizers⁴. 6. The respective values Δc_p are calculated from Eq. (21) for values γ resp. V complying with the previous condition and they are substituted into Eq. (22) for calculation of cross-sectional area and into Eq. (23) for calculation of height of the fluidized bed. 7. Another partial check is made: ratio of the height of the fluidized bed to its diameter is used to be⁵ 1 : 1 up to 2 : 1, only exceptionally even higher. Values which substantially differ from the mentioned limits are not considered. 8. From Eq. (24) is calculated the circulation velocity. 9. For another check serves Eq. (14); the retention time of crystals calculated from it should be⁵ within the limits of 5–15 hours.

II. *The value is known of the system constant B, of the order of kinetic equation of nucleation m and of the kinetic constant of the crystal growth.* 1. Values L_0 and u are determined in the same way as in the previous case, I - 2. 2. A number of degrees of desupersaturation γ is chosen and from Fig. 1 are read off the corresponding values Γ . 3. From Eq. (19) is calculated the volume of the fluidized bed. 4. The check is made according to I - 5. 5. From Eq. (18) are calculated the corresponding values Δc_p . 6. Those of values Δc_p which correspond to the lower supersaturation than is the width of metastable region of the given system, are used for calculation of A and h by use of Eq. (22) and (23). 7. Further calculations are made in the same way as in I - 7 to 9.

III. *Calculation with the choice of working supersaturation, which is suitable especially in the case of known upper limit of supersaturation at which the substance still provides well developed crystals¹.*

1. Calculation is made of L_0 , u and Γ according to II - I and 2. 2. Calculation of the volume of the fluidized bed is made with the use of Eq. (18), the cross-sectional area of the fluidized bed is calculated from Eq. (22) and the height of the fluidized bed from Eq. (23) (the left side). 3. The partial results are checked and selected in the same way as in previous paragraphs. 4. For the choice of the suitable supersaturation for this form of calculation can be used advantageously the equation

$$(\Delta c_p)^n = 3\alpha Q_c CL_p / \beta k_g \tau, \quad (30)$$

which directly results from Eqs (13) and (14) with provision that the residence time of crystals in the crystallizer τ varies often⁵ around the values $\tau = 3 \cdot 10^4$ s.

Comparison of Obtained Relationships with Previously Published Equations for Calculating the Classifying Crystallizers

The first cited work⁴ gives the relation for calculating the residence time of crystals in a classifying crystallizer for conditions $n = 1$, $\gamma = 1$, i.e. only for the simplest case of the crystal growth dependent on the first power of the supersaturation, while at the same time the solution is during the passage through the fluidized bed completely desupersaturated. It was assumed that nucleation did not take place and into the fluidized bed was continuously fed exactly the necessary number of nuclei. The mentioned conditions lead to very high values of the constant C and from that also follows a considerable calculated height of the fluidized bed. As the main shortcoming of this work can be considered the assumption of complete desupersaturation which is theoretically attainable in an infinitely high fluidized bed and which is quite understandably in practice never fulfilled.

Another of the cited works⁵ already represents a considerable generalization of the model; it gives the general differential equation (12), but its solution is made only for $n = 1$ and $n = 2$ and for condition $L_0 \ll L_p$ although for $\gamma = 1$ is given a more general solution. As it follows from now derived relations and especially from Table I, value of the constant C is very much dependent on the value L_0/L_p , especially at low values of this ratio. At the same time, studies on hydrodynamics of fluidized beds^{6,11} show that real values L_0/L_p vary around 0.1 and these are conditions substantially different from the original approximation $L_0 \ll L_p$.

CONCLUSIONS

At the end let us notice some of general conclusions following from individual relations.

Horizontal cross-sectional area of the fluidized bed depends directly on the capacity of the apparatus, indirectly on the upflow velocity of the liquid, value of the inlet supersaturation and on degree of desupersaturation. The dependence on size of product crystals is rather inexpressive. On the other hand, *the height of the fluidized bed* does not depend on the production rate of the apparatus but depends to a great extent on the required size of product crystals, size of the smallest crystals L_0 and on the degree of desupersaturation, and is indirectly proportional to the growth rate constant of crystals. It depends on the supersaturation only in case when $n > 1$ in the way that with increasing supersaturation it decreases.

Volumetric circulation velocity of the liquid depends directly on the production rate of the apparatus, indirectly on the supersaturation and on the degree of desupersaturation; the dependence on the size of product crystals is inexpressive. *Volume of the fluidized bed* is in principle given by the product of its cross-sectional area and of its height so that it depends in a complicated way on a whole number of parameters; expressive is the direct proportionality with the production rate which follows from the corresponding cross-sectional area, and the dependence on the size of product crystals and of the smallest still retained crystals.

The derived relations enable explicit calculation of main dimensions of classifying crystallizers for laboratory measurements. Because of a number of limiting conditions, it is however necessary to check systematically credibility of the calculated data. The presented relations determine simultaneously the type of dependence of main dimensions of the classifying crystallizer on parameters of crystallization and thus they enable the design of models of these apparatuses. For the given relations for example follows decisively that the height of the classified fluidized bed of crystals does not depend on the production rate of the apparatus and that the cross-sectional area is a linear function of the production rate from which follow important conclusions for the design of pilot-plant models as well as for industrial units.

Conclusions made in the previous paper⁵ have been verified also on the precised model and are valid without substantial changes.

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LIST OF SYMBOLS

A	horizontal cross-sectional area of the fluidized bed	$[m^2]$
A_1	surface area of crystals suspended in a mass unit of solution	$[m^2/kg \text{ solvent}]$
B	constant of system, defined by Eq. (20)	$[s^{n/m-1} c^{n-m}]$
C	proportionality constant in Eq. (14)	
c	concentration	$[kg/kg \text{ free solvent}]$
Δc	supersaturation	$[kg/kg \text{ free solvent}]$
Δc_p	supersaturation at the inlet of fluidized bed of crystals	$[kg/kg \text{ free solvent}]$
Δc_0	supersaturation at the outlet of fluidized bed of crystals	$[kg/kg \text{ free solvent}]$
$(\overline{\Delta c})_n$	average supersaturation in respect to the nucleation kinetics	$[kg/kg \text{ free solvent}]$
θ_p	linear crystallization rate at supersaturation Δc_p	$[m/s]$
h	height of fluidized bed	$[m]$
k_n	rate constant of nucleation	$[s^{-1} c^{1-m}]$
k_g	rate constant of crystal growth	$[kg \text{ m}^{-2} s^{-1} c^{-n}]$
L	characteristic length of crystals	$[m]$
L_0	characteristic length of smallest crystals, just still retained in the fluidized bed	$[m]$
L_p	characteristic length of product crystals	$[m]$
L_z	characteristic length of crystal nuclei	$[m]$
m	order of kinetic equation of nucleation	
m_z	mass of crystal nuclei contained in a mass unit of solution	$[kg/kg \text{ solvent}]$
m_c	mass of crystals	$[kg/kg \text{ solvent}]$
n	order of kinetic equation of crystal growth	
N_z	number of crystal nuclei formed in unit of time in unit mass of solution	$[s^{-1} (kg \text{ solvent})^{-1}]$
P	production rate of crystallizer	$[kg/s]$

Q	amount of solution circulated [kg solvent/s]
t	time [s]
u	upflow velocity of solution [m/s]
V	volume of fluidized bed [m ³]
V_Q	volumetric flow rate of circulated solution [m ³ s ⁻¹]
W	weight of crystals retained in the fluidized bed [kg]
W_0	weight of free solvent in the fluidized bed [kg free solvent]
α	volume based shape factor
β	surface based shape factor
γ	relative desupersaturation, defined by Eq. (4)
Γ	function defined by Eq. (8)
ϵ	fluidized bed porosity
$\bar{\epsilon}$	average fluidized bed porosity
ρ_c	density of crystals [kg m ⁻³]
ρ_l	density of solution [kg m ⁻³]
τ	residence time of crystals in crystallizer [s]

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